The contact conductance of a one-dimensional wire partly embedded in a superconductor

Raphael Matthews and Oded Agam
The Racah Institute of Physics, The Hebrew University, Jerusalem, Israel

The conductance through a semi-infinite one-dimensional wire, partly embedded in a superconducting bulk electrode, is studied. When the electron-electron interactions within the wire are strongly repulsive, the wire effectively decouples from the superconductor. If they are moderately or weakly repulsive, the proximity of the superconductor induces superconducting order in the segment of the wire embedded in it. In this case it is shown that the conductance exhibits a crossover from conductive to insulating behavior as the temperature is lowered down. The characteristic crossover temperature of this transition has a stretched exponential dependence on the length of the part of the wire embedded in the superconductor. The amount of this stretch is determined by the nature of the electron interactions within the wire.

I. INTRODUCTION

In recent years one dimensional interacting electron systems have attracted a large amount of attention. Part of the interest in these systems lies in the fact that electron-electron interactions in 1-D wires, even when weak, cannot be considered perturbatively. A question of practical importance, dealt with by a number of authors[1]-[11], is that of the contact conductance of a one dimensional system connected to an external electrode. Most works on the subject picture the junction as a one dimensional wire connected to the electrode at a point. Here it has been found that electron-electron interactions strongly influence the conductance through the junction. In particular, repulsive interactions in the wire drive the system to be insulating at low enough temperatures, unless the contact is perfectly clean. Attractive interactions, on the other hand, mask obstructions at the interface between the two systems. These results do not qualitatively change whether the electrode is superconducting or metallic.

In this paper we explore the behavior of a junction between a 1-D wire and a superconducting electrode of different geometry, specifically, the situation where the wire is embedded some distance into the electrode, as illustrated in Fig. 1. In this case one expects that superconducting order is induced in the part of the wire which is embedded in the superconductor, enhancing the conductance of the junction. On the other hand, for point contacts with even a small amount of normal reflection (and repulsive interactions in the wire), it has been shown that superconducting order suppresses the conductivity of the junction[8,9]. The main goal of this work is to clarify how these two competing effects determine the behavior of the junction as a function of the temperature (or the applied voltage), and length of the embedded wire.

As expected, provided interactions within the wire are not too strong, superconducting order is induced in the part of the wire embedded in the bulk, and an effective gap is formed in the wire whose value is determined by the tunneling rate between the superconductor and the wire, as well as the nature of the electron interactions.

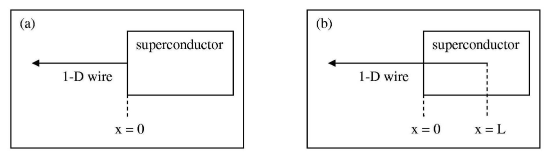

Figure 1: A schematic representation of a 1-D wire in contact with a superconductor: (a) A point contact. (b) A wire embedded a length L into the bulk

Yet, in spite of this proximity effect, the junction conductance exhibits a crossover from a conductive state, governed by Andreev scattering, to an insulating behavior at low enough temperatures, characterized by a power law dependence on the temperature. This behavior results from the finite amount of normal backscattering within the wire. We found that the characteristic crossover temperature between the two regimes has a stretched exponential dependence on the length of the junction, and the amount of stretch is determined by the strength of the electron interactions in the wire.

The article is organized as follows: In the next section the model whereby a 1-D wire is embedded infinitely deep into a bulk BCS superconductor is introduced. By integrating out the superconductor degrees of freedom the effective action of the embedded wire is obtained. In Sec. II and the Appendix, renormalization group (RG) techniques are employed in order to characterize the behavior of a 1-D wire embedded in a BCS superconductor. This RG flow allows one to deduce the low energy properties of the finite part of the wire embedded in the superconductor. In Sec. III the conductance of the wire-superconductor junction is evaluated. Finally, the results are summarized and discussed in Sec. IV.

II. MODEL

We consider a single 1-D wire with interacting electrons (including backscattering), embedded inside a standard BSC superconductor (SC). The action of the system is a sum of three contributions:

$$S = S_{sc}\left(\bar{\varphi},\varphi\right) + S_W\left(\bar{\psi},\psi\right) + S_t\left(\bar{\psi},\psi;\bar{\varphi},\varphi\right) \tag{1}$$

where S_{sc} is the action of the superconductor, S_W is the action of the wire, and S_t describes the tunneling between the two systems. ψ_σ is the electronic field operator in the wire, φ_σ is the field operator in the SC, and σ denotes the spin index.

The action of the SC, modeled by the standard BCS hamiltonian with a constant pairing amplitude Δ_{sc}, has the form

$$S_{sc} = \int d^4\xi \left[\bar{\Phi}(\xi)\begin{pmatrix}\partial_\tau + H_0 & \Delta_{sc}\\ \Delta^*_{sc} & \partial_\tau - H_0\end{pmatrix}\Phi(\xi)\right] \tag{2}$$

where the vector $\xi = (x,y,z,\tau)$ contains three space coordinates and an imaginary time, $\Phi^T = (\varphi_\uparrow, \bar{\varphi}_\downarrow)$ is the SC electron field in Nambu notation, and $H_0 = -\frac{\hbar^2}{2m}\nabla^2 - \mu$ is the free Fermi gas Hamiltonian, with m and μ as the electron mass and chemical potential, respectively.

The 1-D wire is modeled as a Bosonized Luttinger liquid. Since the literature on this subject is quite extensive (ref. [13] and therein), we will only point out the more relevant details to the topic at hand.

We represent the electronic fields[12],

$$\psi_{r,\sigma}(x) = \frac{1}{\sqrt{2\pi\alpha}} e^{irk_f x} e^{-i\sqrt{\frac{\pi}{2}}\left[r\phi_c(x) - \theta_c(x) + \sigma(r\phi_s(x) - \theta_s(x))\right]}, \tag{3}$$

in terms of boson charge and spin density fields: $\partial_x\phi_\nu(x)$, and their conjugates, $\theta_\nu(x)$, where $\nu = c/s$ denotes the charge and spin sectors, respectively. In the above formula, k_f is the Fermi wave number, α is the short distance cutoff, r is the chiral index representing the right ($r = 1$) and left ($r = -1$) moving part of the electronic field, and $\sigma = 1$ for $\uparrow$ spin while $\sigma = -1$ for $\downarrow$ spin.

In terms of the boson fields, the action of the wire becomes a sum of three contributions:

$$S_W = S_c(\theta_c) + S_s(\phi_s) + S_{bs}(\phi_s), \tag{4}$$

where

$$S_c(\theta_c) = \frac{1}{2}\int dx \int_0^\beta d\tau u_c K_c \left(\frac{1}{u_c^2}\left(\partial_\tau\theta_c(x,\tau)\right)^2 + \left(\partial_x\theta_c(x,\tau)\right)^2\right) \tag{5}$$

describes the charge sector (after the dependence on the ϕ_c field has been integrated out[13]). A similar expression describes the spin sector, $S_s(\phi_s)$, with the subscript c replaced by s and K_c replaced by K_s^{-1}. Here K_c, K_s, u_c, and u_s are model specific parameters describing the interaction strength (K) and the mode velocity (u) of the charge and the spin fields. For the noninteracting case $K_c = K_s = 1$ and $u_c = u_s = v_f$, where v_f is the Fermi velocity of the wire. For an interacting system, values of $K_c > 1$ and $K_s < 1$ correspond to attractive interactions, while $K_c < 1$, $K_s > 1$ correspond to repulsive ones. Generally in an interacting system the velocity of the two modes differ, $u_c \neq u_s$.

The third term of the wire's action, S_{bs}, describes backscattering of two electrons with opposite spins. Namely a collision which effectively results in a spin flip between the right and left moving parts of the electronic field ($\sim \psi^\dagger_{L,\sigma}\psi_{R,\sigma}\psi^\dagger_{R,-\sigma}\psi_{L,-\sigma}$). This term has the form

$$S_{bs} = \frac{2g}{(2\pi\alpha)^2}\int dx d\tau \cos\left(\sqrt{8\pi}\phi_s(x,\tau)\right), \tag{6}$$

where g is the backscattering coupling constant.

Finally, the tunneling between the superconductor and the wire is described by

$$\begin{aligned} S_t \; = \; & \int d^4\xi' \int dx d\tau \sum_{\sigma=\uparrow,\downarrow} \bar{\varphi}_\sigma(\xi') t(\xi';x,\tau)\psi_\sigma(x,\tau) \\ & + H.C., \end{aligned} \tag{7}$$

where $\psi_\sigma = \sum_r \psi_{r,\sigma}$, and $t(\xi';x,\tau)$ is the tunneling matrix element. In the simplest case this tunneling is instantaneous, homogeneous in space, short distant and SU(2) spin symmetric. Under these assumptions it takes the form

$$t(\xi';x,\tau) = \tilde{t}(x)\delta(x'-x)\delta(y')\delta(z')\delta(\tau'-\tau) \tag{8}$$

where $\tilde{t}(x)$ has the characteristic function form:

$$\tilde{t}(x) = \begin{cases} t_0 & \text{for } 0 \leq x \leq L \\ 0 & \text{otherwise} \end{cases} \tag{9}$$

where L is the length of the part of the wire embedded in the superconductor (see fig. 1 (b)).

The effective action of a wire embedded in a superconductor is obtained by tracing out the superconductor degrees of freedom,

$$e^{-S_{eff}\left(\bar{\psi},\psi\right)} = \int D[\bar{\varphi},\varphi] e^{-S\left(\bar{\varphi},\varphi;\bar{\psi},\psi\right)} \tag{10}$$

where S is given by (1). Since the action is quadratic in the superconductor field, this integration is straightforward. The result may be written as a sum of three terms, $S_{eff} = S_W + S_{ph} + S_{pp}$, where S_{ph}, and S_{pp} represent contributions associated with Giever (particle-hole) and Andreev (particle-particle) tunneling, respectively. At energies above the superconductor gap Δ_{sc} the particle-hole term is dominant and it's contribution, on integrating it out, will be to renormalize the chemical potential. On the other hand, in the low temperature regime (well below the superconductor gap Δ_{sc}) the particle-hole term vanishes and the main contribution comes from the Andreev tunneling term, S_{pp}. In this limit, after averaging over the rapid spatial oscillations, the tunneling becomes local in space and time. Expressing it in terms of the bosonic fields we thus have

$$S_{eff} = S_W + S_{pp} \tag{11}$$

where S_W is given by (4), and

$$S_{pp} = \frac{2\Delta}{\pi\alpha}\int dx d\tau \cos\left(\sqrt{2\pi}\phi_s(x,\tau)\right)\cos\left(\sqrt{2\pi}\theta_c(x,\tau)\right), \tag{12}$$

where $\Delta \simeq \left(t_0 N_0 \frac{\pi^2}{p_f}\right)^2$. Here N_0 is the normal density of states of the SC at the Fermi level, and p_f is the Fermi momentum.

In terms of the original *fermionic* fields, the tunneling term has the form of the regular pairing term in the standard BCS theory:

$$S_{pp} = \Delta\int dx d\tau \left(\bar{\psi}_\uparrow(x,\tau)\bar{\psi}_\downarrow(x,\tau) + \psi_\downarrow(x,\tau)\psi_\uparrow(x,\tau)\right)$$

In the absence of electron-electron interactions, this term, along with the free quadratic kinetic term of the model can be diagonalized by the standard Bogolubov transformation. The excitation spectrum of this system will be gapped with an energy of Δ.

III. THE RENORMALIZATION GROUP FLOW EQUATIONS

The effective action (11) describes the physics of the junction for a temperature up to the order of the superconductor gap, Δ_{sc} (which will subsequently provide the high energy cut-off of our system).

In order to describe the behavior of the system at much lower energy scales, and take into account the electron-electron repulsive interactions, we shall employ a real space RG approach, following Giamarchi & Schulz[13,14]. As usual in these cases the RG procedure manifest itself in a flow of the coupling constants of the problem as the ultraviolet cut off is reduced from $1/\alpha$ to $1/\alpha'$. In our problem these coupling constants are: The interaction strengths, K_ν; The mode velocities, u_ν; The dimensionless backscattering constant, $y = \frac{g}{\pi u_s}$; And the dimensionless pair tunneling strength $\widetilde{\Delta} = \Delta\frac{\alpha}{u_s}$. The flow equations of these coupling constants (see Appendix for the details of the derivation) are:

$$\begin{aligned}
\frac{dK_c}{dl} &= X_c\left(\frac{u_s}{u_c}\widetilde{\Delta}\right)^2 && (13)\\
\frac{dK_s}{dl} &= -K_s^2\left(X_s\widetilde{\Delta}^2 + \frac{y^2}{2}\right) && (14)\\
\frac{d\widetilde{\Delta}}{dl} &= \widetilde{\Delta}(2 - \frac{1}{2}(K_s + K_c^{-1} + y)) && (15)\\
\frac{dy}{dl} &= y(2 - 2K_s) - 2X_s\widetilde{\Delta}^2 && (16)\\
\frac{du_c}{dl} &= u_c K_c^{-1} W_c\left(\frac{u_s}{u_c}\widetilde{\Delta}\right)^2 && (17)\\
\frac{du_s}{dl} &= u_s K_s W_s \widetilde{\Delta}^2 && (18)
\end{aligned}$$

where $dl = d\log\alpha$ is the dimensionless change in the ultraviolet cutoff,

$$X_{c(s)} = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\left(\cos^2(\varphi) + \left(\frac{u_{s(c)}}{u_{c(s)}}\right)^2\sin^2(\varphi)\right)^{\frac{-g_{s(c)}}{2}} \tag{19}$$

$$W_{c(s)} = \frac{1}{2\pi}\int_0^{2\pi} d\varphi\cos(2\varphi)\times\left(\cos^2(\varphi) + \left(\frac{u_{s(c)}}{u_{c(s)}}\right)^2\sin^2(\varphi)\right)^{\frac{-g_{s(c)}}{2}} \tag{20}$$

and $g_c = K_c^{-1}$ while $g_s = K_s$.

We shall restrict our analysis to the spin symmetric case (i.e. the situation where the interactions between electrons with parallel and opposite spins are identical). In this case[14] $K_s \simeq 1 + \frac{y}{2}$, and equations (14) and (16) for the dimensionless backscattering y and spin interaction strength K_s reduce to:

$$\frac{dy}{dl} = -y^2 - 2X_s\widetilde{\Delta}^2, \tag{21}$$

thus maintaining spin invariance.

A. Analysis of the Flow Equations

The above RG equations imply that the charge and spin velocities, u_c and u_s, renormalize towards each other. This is a consequence of the correlations between spin and charge excitations generated by the proximity effect. From Eqs. (17), (18), and (20) one can observe that the signs of W_c and W_s are determined by the ratio u_c/u_s in such a way that the velocities approach each other.

Another consequence of the RG equations,(13) and (14), is that K_c can only grow, while K_s can only be reduced. This flow stem from the finite value of the tunneling parameter $\widetilde{\Delta}$ (and y). Thus the RG behavior of $\widetilde{\Delta}$ controls the behavior of the system.

The separatix between the regions where $\widetilde{\Delta}$ is relevant or irrelevant can be obtained numerically for a given set of bare parameters. It is clear that for initially repulsive interactions ($K_s^{(0)} > 1,\ y^{(0)} > 0,\ K_c^{(0)} < 1$), $\widetilde{\Delta}$ will be relevant if:

$$4 - (K_s^{(0)} + \left(K_c^{(0)}\right)^{-1} + y^{(0)}) > 0, \tag{22}$$

since the flow equations can only drive $K_{c/s}$ to be more "attractive". In Fig. 2 we depict the RG flow in the $\widetilde{\Delta}$-K_c plane, in the spin symmetric case, for several initial values of K_c^0 in the range between 0.32-0.325. Fig. 3 shows the separatix between relevant and irrelevant tunneling for initial bare (repulsive) values of K_c and y. One can observe that K_c must be smaller than $\frac{1}{3}$ for the tunneling to be irrelevant.

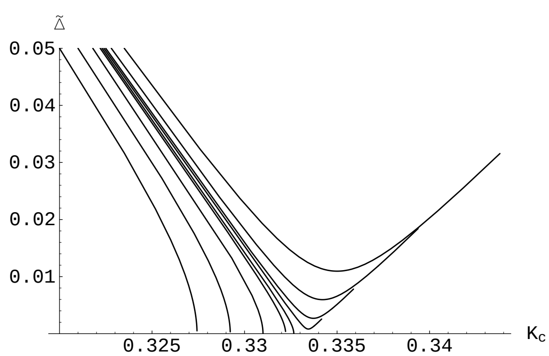

Figure 2: Plots of the flow of $\widetilde{\Delta}$ as a function of K_c for initial values of $K_c^{(0)}$ between 0.32-0.325. The transition between irrelevant ($\widetilde{\Delta} \to 0$) and relevant ($\widetilde{\Delta} \to \infty$) tunneling occurs at $K_c^{(0)} \sim 0.3223$. The initial values of $\widetilde{\Delta}^{(0)}$ and $y^{(0)}$ are 0.05 and 0.1 respectively

In the case where $\widetilde{\Delta}$ is irrelevant, the wire effectively decouples from the superconductor. The pair tunneling between the two systems is suppressed and the superconductor-wire junction becomes insulating. Notice however that this behavior takes place at very strong repulsive interactions, $K_c < \frac{1}{3}$, where the system tends to Wigner crystallize[13].

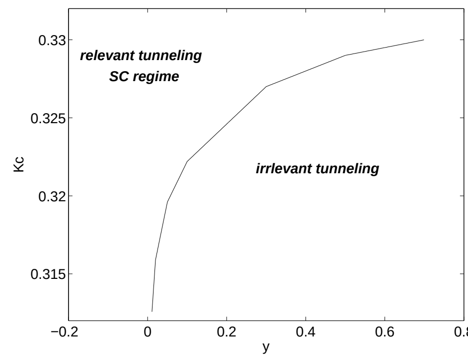

Figure 3: The curve describes the separatix between relevant tunneling, where the system flows to a superconducting state, and irrelevant tunneling, where the system flows to a decoupled wire, as a function of the bare values of K_c and y

In the situation where $\widetilde{\Delta}$ is relevant, the system flows to a singlet superconducting state. The interaction parameters become attractive and spin-charge separation is no longer valid. Yet, before the interaction parameters obtain their asymptotic values (i.e $K_c \to \infty$, $K_s \to 0$ and $y \to -\infty$ which correspond to "infinitely" attractive interactions), the RG equations (13- 18) will cease to be valid since the small parameters of our perturbation theory, $\widetilde{\Delta}$ and y, will flow to the strong coupling regime.

B. Length Dependance of the Effective Gap

In what follows we consider a weakly interacting Luttinger liquid, where the bare interaction parameters are close to unity. Moreover, in order to ensure that the perturbative RG equations (13- 18) remain valid we shall assume that L, the embedding length, is of order or smaller then the bare superconducting correlation length, v_f/Δ. Since L serves as an infrared cutoff for the RG flow, this condition ensures that the this flow is confined to the perturbative regime. We shall also assume that the temperature is much smaller than Δ, and approximate the velocities of the spin and charge sectors by their asymptotic renormalized values: $u_c = u_s = v_f$.

In the limit of weak interactions the RG equations for $K_{s/c}$ are of second order in the perturbation parameters ($\widetilde{\Delta} \ll 1, y \ll 1$), while the equation for $\widetilde{\Delta}$ (Eq. 15) is of first order (neglecting the y dependance, which is of second order as well). The equation for the coupling y (Eq. 16) is also second order in small parameters, at least if we consider the spin symmetric case . Thus one can assume K_c, K_s, and y to be approximately constants, and consider the simplified equation for $\widetilde{\Delta}$:

$$\frac{d\widetilde{\Delta}}{dl} = \gamma\widetilde{\Delta}, \tag{23}$$

where

$$\gamma = 2 - \frac{1}{2}(K_s + K_c^{-1}). \tag{24}$$

Integrating the above equations from $l = 0$ to $l = \log(L/\alpha)$, and using the relation $\tilde{\Delta} = \Delta\alpha/v_f$, we obtain the renormalized value of the gap of the wire embedded in the superconductor:

$$\Delta_{eff} = \Delta \left(\frac{L}{\alpha}\right)^{\gamma-1}. \tag{25}$$

In particular, repulsive interactions ($K_c < 1$ and $K_s > 1$, and therefore $\gamma - 1 < 0$) reduce the effective gap in the wire.

This renormalization of the gap implies that the effective correlation length,

$$\xi_{eff} = \frac{v_f}{\Delta_{eff}} = \xi \left(\frac{L}{\alpha}\right)^{1-\gamma} \tag{26}$$

is larger than the bare correlation length $\xi = v_f/\Delta$.

The above results are valid as long as the RG flow stays within the perturbative regime, namely $\tilde{\Delta} < 1$. This condition implies that L should be shorter than $\xi(\xi/\alpha)^{\frac{1}{\gamma}-1}$.

IV. THE WIRE-SUPERCONDUCTOR JUNCTION

At this stage of the RG procedure (pursued up to the scale L) the junction between the wire and the superconductor may be assumed to be point-like. Thus if the temperature (or the applied voltage) is smaller than $\hbar v_f/L$ one may continue to integrate out the high energy degrees of freedom in the part of the wire which is not embedded into the superconductor down to the relevant energy scale. This may be achieved following the procedure described in the literature[9]. The important ingredient, now, is the magnitude of normal back scattering from the junction. The latter is of order of $r_N \simeq r_0 e^{-2L/\xi_{eff}}$ where r_0 is system specific reflection amplitude in the absence of superconductivity, while ξ_{eff} is the correlation length (26) within the part of the wire embedded in the superconductor. This behavior of r_N results form the fact that only the charge that is not converted to the condensate backscatters from the edge of the wire embedded in the superconductor. According to[5] the normal current reduces exponentially with the distance on a length scale of ξ , which gives the above estimate for r_N. Now, since ξ_{eff} depends on L, the magnitude of the normal reflection has a stretched exponential dependence:

$$r_N \simeq r_0 e^{-aL^\gamma} \tag{27}$$

where $a = 2\alpha^{1-\gamma}/\xi$, and the amount of stretch, γ, is dictated by the nature of the electron-electron interactions within the wire, as follows from (24).

The behavior of the backscattering, clearly, manifests itself in the conductance of the junction. In the limit of sufficiently high temperatures, one obtains[8]:

$$G \;=\; G^0_{NS} - \delta G, \tag{28}$$

where $G^0_{NS} = 2G^0_{NN} = 4\frac{e^2}{2\pi\hbar}$ represents the conductance of an ideal junction where only perfect Andreev reflections take place, and

$$\delta G \;\propto\; r_0^2 e^{-2aL^\gamma}\; T^{-2(1-K_c)}. \tag{29}$$

The above formula holds for the range of temperature where $\delta G \ll G^0_{NS}$, since δG cannot be larger than G^0_{NS}. Nevertheless, it has been shown[6]-[10], that any scatterer, at a point contact between a wire with repulsive interactions and a SC, will eventually drive the conductance to zero as the temperature is lowered. Since any finite length junction will have some backscattering, the conductance should drop to zero for low enough temperatures, as illustrated in Fig. 4. The crossover temperature, T^* from the conductive and the insulating behavior of the junction depends on the length of the wire embedded in the superconductor and its scaling behavior may be deduced from (28) and (29):

$$T^* \propto e^{-\frac{a}{1-K_c}L^\gamma} \tag{30}$$

This result implies that the temperature scale at which the effects of backscattering becomes substantial reduce as a stretched exponent with the length of the junction, and the stretch is determined by the interactions, through the parameter γ.

.

V. SUMMARY AND CONCLUSIONS

In this work we studied a junction of a 1-D wire embedded a certain length, L, into a bulk superconductor. We first characterized the nature of the contact between the 1-D wire and the superconductor using a real space RG scheme. We found that repulsive interactions in the wire compete against the superconducting order being imposed by the bulk superconductor. The system can flow to either of two phases, depending on the nature of the interactions. When the interactions are strongly repulsive the tunneling between the two systems becomes irrelevant and the wire essentially decouples from the bulk superconductor. For moderate repulsive interactions, and for attractive interactions, tunneling is relevant, and the bulk superconductor induces superconducting order in the wire. The gap opened in the wire depend on the tunneling strength, and electron-electron interactions modify its nominal value.

The finite length of the part of the wire embedded in the superconductor, L, implies that the RG flow, in general, does not reach its asymptotic (non-perturbative) limit. Thus L introduces itself in the behavior of the effective gap, and the effective correlation length, in the

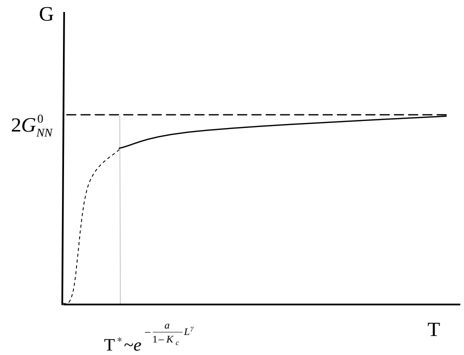

Figure 4: A schematic graph of the conductance as a function of temperature for repulsive interactions in the wire. The decay of the conductance depends on the length of the junction as a stretch exponent. The longer the junction, the lower the temperature at which the decay sets in. In the limit of weak backscattering, represented by the solid curve, we can estimate the temperature dependance of the conductance through Eq. (29). In the zero temperature limit, represented by the dashed curve, any initial finite backscattering will eventually drive the conductance to zero. The crossover temperature, Eq. (30) between the two behaviors is estimated to scale as a stretched exponent in the length of the junction.

wire. This, in turn, dictates a stretched exponential behavior of the normal reflection from the junction as function of L. For energy scales (temperature or voltages) beneath the effective gap, we described the qualitative picture of the conductance as a function of temperature and length of the embedded segment of wire, see Fig. 4.

Several simplifications have been used for our analysis: One is that our model treats a semi-infinite wire with a single junction, while in practical situations a finite wire is usually connected to two reservoirs. This idealization holds as long as the segment of the wire outside the superconductor is long enough compared to $\hbar v_f/T$, where v_f is the Fermi velocity and T is the temperature. Additional simplification is the assumption that the embedding of the wire into the bulk superconductor does not introduce inhomogeneities, i.e. the wire can be still considered to be clean, and that the tunneling to the superconductor is homogenous along the wire. This approximation holds when the transport mean free path in the wire is longer than L. In the opposite limit one expects a different behavior of the proximity effect which will change the conductance of the system.

The authors gratefully acknowledge discussions with Dror Orgad. This work has been supported in part by the Israel Science Foundation (ISF) funded by the Israeli Academy of Science and Humanities, and by the USA-Israel Binational Science Foundation (BSF).

VI. APPENDIX

The mathematical formulation used in this work follows closely the real space RG procedure used by Giamarchi & Schulz[13,14]: In this procedure one evaluates a correlation function in the wire, of the form (the time ordering symbol is suppressed):

$$R_\varphi(x_a,\tau_a;x_b,\tau_b) = \left\langle e^{i\gamma\sqrt{2\pi}(\varphi(x_a,\tau_a)-\varphi(x_b,\tau_b))}\right\rangle, \quad (31)$$

where φ can symbolize any of the boson fields and γ is some constant. For the following discussion it will suffice to examine only one of the sectors, for instance the spin sector. The same considerations can be carried on straightforwardly for the charge sector.

The fact that the relevant boson fields in the spin sector is ϕ_s leads naturally to the evaluation of the correlation functions:

$$R_{\phi_s}(x_a,\tau_a;x_b,\tau_b) = \left\langle e^{i\gamma\sqrt{2\pi}(\phi_s(x_a,\tau_a)-\phi_s(x_b,\tau_b))}\right\rangle. \quad (32)$$

Unfortunately this correlation function cannot be calculated exactly using the complete effective action (11). Though, if the tunneling and backscattering parameters, (t and g respectively), are small then it may be computed perturbatively. To second order in these parameters this function is found to be:

$$R_s(\vec{r}_{a,b}) = e^{-\gamma^2\left(K_s^{eff}F_s(\vec{r}_{a,b})+D_s^{eff}\sin^2(\varphi_{\vec{r}_{a,b},s})\right)}, \quad (33)$$

where $\vec{r}_{a,b}=\vec{r}_a-\vec{r}_b$, and $\varphi_{\vec{r},s}$ is the angle between the vector $\vec{r}=(x,u_s\tau)$ and the x axis. The function F_s is (at zero temperature)[13]:

$$F_s(x,\tau) = \frac{1}{2}\ln\left(\frac{x^2+(u_s|\tau|+\alpha)^2}{\alpha^2}\right). \quad (34)$$

Apart from the term proportional to $\sin^2(\varphi_{\vec{r}_{a,b},s})$, this functional form is identical to the free correlation function:

$$R_s^{(0)}(\vec{r}_a-\vec{r}_b) = e^{-\gamma^2K_sF_s(\vec{r}_a-\vec{r}_b)}, \quad (35)$$

but with an effective Luttinger interaction constant K_s^{eff} modified by the perturbations:

$$K_s^{eff} \;=\; K_s - K_s^2\left(\widetilde{\Delta}^2X_s\int_\alpha^\infty\frac{dr}{\alpha}\left(\frac{r}{\alpha}\right)^{3-K_s-K_c^{-1}} + \frac{y^2}{2}\int\frac{dr}{\alpha}\left(\frac{r}{\alpha}\right)^{3-4K_s}\right). \quad (36)$$

Here $\widetilde{\Delta}=\frac{\Delta\alpha}{u_s}$ is the dimensionless tunneling parameter, $y=\frac{g}{\pi u_s}$ is the dimensionless spin backscattering parameter, and X_s is a geometrical term given by:

$$X_s = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \left(\cos^2(\varphi) + \left(\frac{u_c}{u_s}\right)^2 \sin^2(\varphi)\right)^{\frac{-K_c^{-1}}{2}} \quad (37)$$

Integrating out high energy degrees of freedom, near the ultraviolet cutoff, corresponds to integrating out a "small ring" between $\alpha \to \alpha' = \alpha + d\alpha$, where α is the small distances parameter of the model. After integration and rescaling, an infinitesimal change is generated in the expressions for K_s^{eff}. In order to keep K_s^{eff} constant with the reduction of the cutoff, it is required that the bare parameters change. For instance, we find that:

$$K_s(\alpha') = K_s(\alpha) - K_s^2(\alpha)\left(\widetilde{\Delta}^2(\alpha)X_s(\alpha) + \frac{y^2(\alpha)}{2}\right)\frac{d\alpha}{\alpha} \quad (38)$$

which generates an (exact) differential equation for K_s:

$$\frac{dK_s}{dl} = -K_s^2(l)\left(X_s\widetilde{\Delta}^2(l) + \frac{y^2(l)}{2}\right) \quad (39)$$

(here $dl = \frac{d\alpha}{\alpha}$).

In a similar fashion one obtains differential equations for the parameters $\widetilde{\Delta}$ and y:

$$\begin{aligned} \frac{d\widetilde{\Delta}^2}{dl} &= \widetilde{\Delta}^2(4 - K_s - K_c^{-1}), && (40)\\ \frac{dy^2}{dl} &= y^2(4 - 4K_s). && (41) \end{aligned}$$

The $\sin^2(\varphi_{\vec{r}_{a,b},s})$ contribution to the correlation function arises from the fact that the tunneling perturbation couples the spin and charge sectors, which were uncoupled without this term. Mathematically, this term characterizes the anisotropy between the space (x) and time ($u_s\tau$) directions. It's pre-factor D^{eff} is given by:

$$D_s^{eff} = D_s + K_s^2\widetilde{\Delta}^2 W_s \int_\alpha^\infty \frac{dr}{\alpha}\left(\frac{r}{\alpha}\right)^{3-K_s-K_c^{-1}}, \quad (42)$$

where W_s is another geometric term factor:

$$W_s = \frac{1}{2\pi}\int_0^{2\pi} d\varphi \cos(2\varphi)\left(\cos^2(\varphi) + \left(\frac{u_c}{u_s}\right)^2 \sin^2(\varphi)\right)^{\frac{-K_c^{-1}}{2}}.$$

Applying the same renormalization scheme to D^{eff}, will generate the flow of the parameter D_s:

$$\frac{dD_s}{dl} = K_s^2(l)W_s(l)\widetilde{\Delta}^2(l). \quad (43)$$

It should be noted that D_s is initially zero but is generated under renormalization.

The parameter D_s controls the renormalization of the velocity parameter. As long as the space and time directions are isotropic the velocity parameter does not flow under renormalization, but they should flow in the anisotropic case. Indeed, assuming that initially the correlation function is described by the function $K_sF_s(\vec{r}_{a,b})$, then a small change of du_s will generate the term: $\frac{K_s}{u_s}\sin^2(\varphi_{\vec{r}_{a,b},s}) \cdot du_s$. Up to a factor, this is exactly the anisotropy term. Therefore, the renormalization of D_s is equivalent to that of the velocity u_s by the following relation (e.g. Eq. 33): $\frac{du_s}{dl} = \frac{u_s}{K_s}\frac{dD_s}{dl}$.

The flow equations for the charge sector can be obtained by an identical procedure. The equations obtained are:

$$\begin{aligned} \frac{dK_c}{dl} &= = X_c(l)\left(\frac{u_s}{u_c}\widetilde{\Delta}(l)\right)^2, && (44)\\ \frac{dD_c}{dl} &= K_c^{-2}(l)W_c(l)\left(\frac{u_s}{u_c}\widetilde{\Delta}(l)\right)^2. && (45) \end{aligned}$$

Finally these equations cannot be exactly correct, since they do not maintain the spin invariance SU(2) of a model that was spin invariant to begin with. (A spin symmetric model is one where the interactions between electrons of opposite and parallel spin are identical). Since the perturbations do not break this symmetry, something in the above result is insufficient. Indeed, it turns out that the remedy for this problem lies in the inclusion of the third order terms of perturbation theory. This correction is presented in reference[14]. It affects only the equations for $\widetilde{\Delta}$ (Eq. (40))and y (Eq. (41)) which become:

$$\begin{aligned} \frac{d\widetilde{\Delta}}{dl} &= \widetilde{\Delta}(2 - \frac{1}{2}(K_s + K_c^{-1} + y)), && (46)\\ \frac{dy}{dl} &= y(2 - 2K_s) - 2X_s\widetilde{\Delta}^2. && (47) \end{aligned}$$

[1] C. L. Kane and M. P. A. Fisher, Phys. Rev B **46**, 15233 (1992).
[2] A. Furusaki and N. Nagaosa, Phys. Rev B **47**, 4631 (1993).
[3] D. Yue, L. I. Glazman and K. A. Matveev, Phys. Rev. B **49**, 1966 (1994).
[4] I. Safi and H.J. Schulz, Phys. Rev. B **52**, R17040 (1995); D.L. Maslov and M. Stone, Phys. Rev. B **52**, R5539 (1995); Y. Oreg and A. M. Finkel'stien, cond-mat/9607149 v1 (1996).
[5] G. E. Blonder, M. Tinkham and T.M. Klapwijk, Phys. Rev. B **25**, 4515 (1982)
[6] R. Fazio, F. W. J. Hekking and A. A. Odintsov, Phys. Rev. Lett. **74**, 1843 (1995).
[7] D. L. Maslov, M. Stone, P. M. Goldbart and D. Loss, Phys. Rev. B **53** 1548 (1996)
[8] Y. Takane and Y. Koyama, J. Phys. Soc. Japan **65**, 3630 (1996).
[9] Y. Takane and Y. Koyama, J. Phys. Soc. Japan **66**, 419

(1997).
[10] I. Affleck, J. S. Caux and A. M. Zagoskin, Phys. Rev. B 62 **1433** (2000).
[11] S. Vishveshwara, C. Bena, L. Balents and M. P. A. Fisher, Phys. Rev. B **66**, 165411 (2002).
[12] There are actually quite a few different conventions used in the bosonization literature. Apart from a factor of $\sqrt{\pi}$ in the normalization of the bosonic fields, we follow the one used by[13].
[13] T. Giamarchi, *Quantum Physics in One Dimension*. Clarendon Press, Oxford (2004).
[14] T. Giamarchi and H. J. Schulz, Phys. Rev. B **37**, 325 (1988).