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TRANSPORT IN LUTTINGER LIQUIDS WITH STRONG IMPURITIES

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The tunnel current of a Luttinger liquid with a finite density of strong impurities is calculated using an instanton approach. For very low temperatures T or electric fields E the (nonlinear) conductivity is of variable range hopping (VRH) type as for weak pinning. For higher temperatures or fields the conductivity shows power law behavior corresponding to a crossover from multi- to single-impurity tunneling. For even higher T and not too strong pinning there is a second crossover to weak pinning. The determination of the position of the various crossover lines both for strong and weak pinning allows the construction of the global regime diagram.

Keywords: Luttinger liquids; disorder; transport

1. Introduction

1D electron systems exhibit a number of peculiarities which destroy the familiar Fermi-liquid behavior known from higher dimensions. Main reason is the geometrical restriction of the motion in 1D where electrons cannot avoid each other. As a consequence excitations are plasmons similar to sound waves in solids. The corresponding phase is called a Luttinger liquid (LL) ^{1,2}. Renewed interest in LLs arises from progress in manufacturing narrow quantum wires with a few or a single conducting channel. Examples are carbon nanotubes ³, polydiacetylen ⁴, quantum Hall edges ⁵ and semiconductor cleave edge quantum wires ⁶.

From a theoretical point of view 1D quantum wires allow the investigation of the interplay of interaction and disorder effects since short range interaction can be treated already within a harmonic bosonic theory ⁷. Central quantity is the interaction parameter K which plays the role of a dimensionless conductance of a clean LL ^{8,1}. The effect of disorder on transport in LLs has been so far considered in two limiting cases:

- (i) The effect of a *single impurity* was considered in ^{8,9,10,11}. Here the conduc-

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tance depends crucially on K . Impurities are irrelevant for attractive ($K > 1$) and strongly relevant for repulsive interaction ($K < 1$), respectively. For finite voltage V and $K < 1$, the conductance is $\sim V^{\frac{2}{K}-2}$ ⁸. These considerations can be extended to two impurities. Depending on the applied gate voltage, Coulomb blockade effects may give rise to resonant tunneling^{8,10}.

(ii) In the opposite case of a *finite density of weak impurities*, (Gaussian) disorder is a relevant perturbation for $K < 3/2$ leading to the localization of electrons. For weak external electric field E the conductivity is highly nonlinear: $\sigma(E) \sim e^{-c/\sqrt{E}}$ ^{12,13,14,15}. At low but finite temperatures T this result goes over into the VRH expression for the linear conductivity $\sigma \sim e^{-c'/\sqrt{T}}$ ^{13,14,15,16,17}. At higher temperatures there is a crossover to $\sigma \sim T^{2-2K}$ ¹⁸.

On the contrary, much less is known in the case of a *finite density of strong pinning centers*^{19,2} which we will address in the present paper. In particular we determine both the temperature and electric field dependence of the (nonlinear) conductivity for this case in a broad temperature and electric field region. The main results of the paper are the conductivities (5), (6), (7) and (8) as well as the crossover behavior summarized in Fig. 1.

2. Model and Instantons

Starting point of our calculation is the action of interacting electrons subject to an external uniform electric field E and strong pinning centers. In bosonized form the action takes the form

$$S = \frac{\hbar}{2\pi K} \int_0^L \int_0^{\lambda_T} dx dy \left\{ (\partial_y \varphi)^2 + (\partial_x \varphi + f x)^2 - \sum_{i=1}^N u \delta(x - x_i) \cos(2\varphi + 2k_F x_i) \right\} \quad (1)$$

The phase $\varphi(x)$ is related to the electron density $\rho(x) = \pi^{-1}(k_F + \partial_x \varphi)(1 + 2 \cos(2\varphi + 2k_F x))$. k_F is the Fermi wave vector, $\tau = y/v$ and $f = FK/v\hbar$. v and $\lambda_T = \hbar v/T$ denote the plasmon velocity and the thermal de Broglie wave length, respectively.

The phase field between the impurities can now be easily integrated out leaving only its values $\varphi(x_i, y) \equiv \phi_i(y)$ at the impurity sites x_i which are assumed to be randomly distributed. The action can then be expressed in terms of Fourier components $\phi_i(y) = \lambda_T^{-1} \sum_{\omega_n} \phi_{i,\omega_n} e^{-i\omega_n y}$, $\omega_n = 2\pi n/\lambda_T$. Thus

$$S = \frac{\hbar}{2\pi K} \sum_{i=0}^N \left\{ \sum_{\omega_n} \frac{\omega_n}{\lambda_T} \left(\frac{|\phi_{i+1,\omega_n} - \phi_{i,\omega_n}|^2}{\sinh \omega_n a_i} + (|\phi_{i,\omega_n}|^2 + |\phi_{i+1,\omega_n}|^2) \tanh \frac{\omega_n a_i}{2} \right) - f(a_{i-1} + a_i) \phi_{i,0} + u_{\text{eff}} \int dy \left[1 - \cos(2\phi_i(y) + 2\pi\alpha_i) \right] \right\} \quad (2)$$

where $a_i = x_{i+1} - x_i$ and $\alpha_i = k_F x_i/\pi$. Since $k_F a_i \gg 1$ below we will assume the α_i to be random phases but keep the impurity distance a_i approximately constant $a_i \approx a$.

Next we consider the current resulting from tunneling processes between metastable states, assuming strong pinning and weak quantum fluctuations, i.e. $K \ll 1$. The tunneling process starts from a classical *metastable* configuration $\tilde{\phi}_i$ which minimizes the impurity potential for all values of y , $E = 0$. Hence $\tilde{\phi}_i = \pi(n_i - \alpha_i)$ where n_i is integer. Among the many metastable states there is one (modulo π) zero field *ground* state $\tilde{\phi}_i^0$ where $n_i = n_i^0 = \sum_{j \leq i} [\Delta\alpha_{j-1}]_G$ ¹³. Here $\Delta\alpha_j = \alpha_{j+1} - \alpha_j$ and $[\alpha]_G$ denotes the closest integer to α . A new metastable state follows from the ground state by adding integers $q_i = \pm 1$ to the n_i^0 .

Next we consider an *instanton* configuration which connects the original state $\tilde{\phi}_i$ with the new state $\tilde{\phi}_i + \pi$, n_i depends in general on y . To be specific, we assume a double kink configuration for the instanton at each impurity site: $\phi_i(y) = \tilde{\phi}_i + \pi$, for $|y - y_i| < D_i - d$, and $\phi_i(y) = \tilde{\phi}_i$, for $|y - y_i| > D_i + d$, with a linear interpolation between the two values at the kink walls in the regions $||y - y_i| - D_i| < d$. $y_i \pm D_i$ is the kink/anti-kink position, $d \sim 1/u$ is the approximate width of the kinks and $2D_i$ their distance. It is plausible that in the saddle point configuration all y_i will be the same, an approximation we will use in the following. With $z_i = \pi D_i/a$ the instanton action can then be rewritten as

$$S_I \approx \frac{2\hbar}{K} \sum_i \left\{ \frac{\Delta\tilde{\phi}_i}{\pi} (z_{i+1} - z_i) - f a^2 z_i + s + \ln \left[\frac{\cosh((z_{i+1} - z_i)/2)}{\cosh((z_{i+1} + z_i)/2)} \tanh \frac{z_i}{2} \cosh z_i \right] \right\} \quad (3)$$

where the sum goes only over impurities with $z_i > 0$. s is a constant that includes the core action of a kink and an anti-kink: $s = \ln(Cau) \gg 1$, where $\ln C/K \gg 1$.

For a given initial metastable state $\{\tilde{\phi}_i\}$, S_I is a function of the variational parameters $\{D_i; i = 1, \dots, N\}$. The nucleation rate Γ and hence the current I is given by $I \propto \Gamma \propto \prod_{i=0}^N \int_0^{i\infty} dD_k \exp(-S/\hbar)$. Here we employ an approximate treatment in which we assume $D_i \equiv D = az/\pi$ for $k < i \leq k + m$ and $D_i = 0$ elsewhere, i.e. tunneling is assumed to occur simultaneously through m neighboring impurities. The instanton is then a rectangular object with extension ma and $2D$ in x and y direction, respectively. The instanton action can then be written as

$$S_{\text{inst}} = \frac{2\hbar}{K} \left\{ z\sigma_m(k) + \ln(1 + e^{-2z}) + m \left(s + \ln \tanh \frac{z}{2} - z \frac{E}{E_a} \right) \right\}. \quad (4)$$

Here we introduced the dimensionless field strength $fa^2/\pi = E/E_a$ where $E_a = 1/(\kappa a^2)$, $\kappa = K/\pi\hbar v$ denotes the compressibility. $\sigma_m(k) = (\nu_k(1) + \nu_{k+m}(-1))/2$ plays the role of a surface tension of the vertical boundaries of the instanton where $\nu_k(q) = q^2 - 2q(\Delta\alpha_k - [\Delta\alpha_k]_G)$. In the ground state $\sigma_m(k)$ is equally distributed in the interval $0 \leq \sigma_m(k) < 2$ ¹⁴. The second and the third contribution in (4) result from the horizontal boundaries of the instanton and include their surface tension s/a and their attractive interaction. The last term describes the volume contribution resulting from the external field.

In addition, we have to include a small dissipative term $S_{\text{bath}} = \frac{2\hbar}{K} m\eta \ln z$, $\eta \ll 1$, in the action in order to allow for energy dissipation¹⁴. However, we will

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omit η -dependent terms in all results where they give only small corrections (apart from possible pre-exponential factors which we do not consider).

A necessary condition for tunneling is $\partial S_{\text{inst}}/\partial z < 0$ for $z \rightarrow \infty$, i.e. $\sigma_m(k) < mE_a/E$. The tunneling probability follows from the saddle point value of the instanton action where z fulfils the condition $\sigma_m(k) - m\frac{E}{E_a} + \tanh z - 1 + \frac{m\eta}{z} + \frac{m}{\sinh z} = 0$.

3. Results and Conclusions

We discuss now several special cases: (i) For sufficiently large fields $E \gg E_a$ the saddle point is $z_s \approx \frac{E_a}{E} \ll 1$ which gives a tunneling probability $\Gamma \propto (E/E_a)^{\frac{2m}{K}-1} e^{-2ms/K}$. The exponent -1 results from the integration around the saddle point. Because of small K and correspondingly large kink core action, tunneling through single impurities ($m = 1$) is preferred and hence the nonlinear conductivity is given by

$$\sigma(E) \sim (E/E_a)^{\frac{2}{K}-2} e^{-2s/K}, \quad E_a < E < E_{1,\text{cr}} = E_a e^s, \quad (5)$$

in agreement with previous results for tunneling through a single weak link ⁸ if we identify $e^{-s/K} \sim t$ with the hopping amplitude t through the link. The upper field strength for the validity of this result can be estimated from $D_s u \equiv z_s a u < 1$ since the instantons loose then their meaning. Using $u \rightarrow u_{\text{eff}} \approx k_F (u/k_F)^{1/(1-K)}$ we find $E_{1,\text{cr}} \sim (k_F/e\kappa a)(u/k_F)^{1/(1-K)}$, which can be also read off directly from (5) as $E_{1,\text{cr}} \sim E_a e^s$. Classically ($K = 0$), $E_{1,\text{cr}}$ corresponds to the case when the field energy Ea the electron gains by moving to the next impurity is smaller than the pinning energy u/κ .

At finite temperatures there is a crossover to a temperature $T \approx EaK$ dependent conductivity

$$\sigma(T, E) \propto \left(\frac{T}{T_a}\right)^{\frac{2}{K}-2} e^{-2s/K} \sinh\left(\frac{Ea}{T}\right) \frac{T}{Ea}, \quad EKa < T < T_{1,\text{cr}} = T_a e^s \quad (6)$$

when the instanton extension $2D_s$ reaches λ_T , i.e. for $E < EaT/T_a$. For temperatures higher than $T_{1,\text{cr}}$ isolated impurities are weak. Following the arguments of ¹⁹ one expects in this region $\sigma \sim T^{2-2K}$.

(ii) In the opposite case of weak fields, $E \ll E_a$, tunneling happens simultaneously through many impurities and the saddle point is $z_s \gg 1$. In this case we can estimate the typical surface tension as $\sigma_k(m) \approx 1/m$ for a chosen pair of sites k and $k + m$, respectively ¹⁴. For very large values of m we can treat m as continuous and the saddle point condition gives $m_s \approx \sqrt{E_a/E} \gg 1$ and $z_s \approx sE_a/(2E)$. The tunneling probability and hence the current is proportional to

$$I \sim \sigma(E) \sim e^{-\frac{2s}{K} \sqrt{E_a/E}}, \quad E < E_a. \quad (7)$$

If we write the result in the VRH form ¹⁶ $I \sim e^{-2ma/\xi_{\text{loc}}}$ we can identify the localization length $\xi_{\text{loc}} \approx aK/s$ of the tunneling charges. There is a crossover to

a temperature dependent conductivity if $\lambda_T < 2D_s$, i.e. for $E < sTE_a/T_a < E_a$ where

$$\sigma(T) \sim e^{-\frac{2}{K}\sqrt{csT_a/T}} \sinh\left(\frac{Ea}{T}\right) \frac{T}{Ea}, \quad EKa/s < T < T_a/s \quad (8)$$

Results (7) and (8) are in agreement with those obtained for weak pinning^{13,14}. (iii)

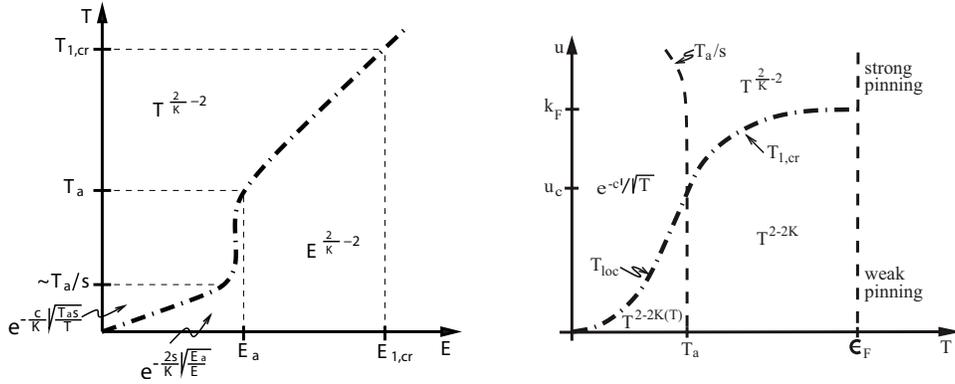


Fig. 1. Left: Field and temperature dependence of the conductivity in the various regions of the $T-E$ plane. T_a , $T_{1,cr}$, s , E_a and $E_{1,cr}$ are explained in the text. The region $T_a > T > T_a/s$, $E < E_a$ is characterized by activated behavior $\sigma \sim e^{-T_a/T} \sinh(\frac{Ea}{T}) \frac{T}{Ea}$. Right: $u-T$ phase diagram of the linear conductivity of disordered LLs. For strong pinning, $u > u_c \sim k_F (k_F a)^{K-1}$ and $T < T_{1,cr} \sim T_a (u/u_c)^{1/(1-K)}$, T_a/s separates the VRH from the single impurity hopping regime. For $T > T_{1,cr}$ impurities become weak. For weak pinning, $u < u_c$, $T_{loc} \sim T_a (u/u_c)^{2/(3-2K)}$ separates VRH from renormalized power law behavior. For $T > T_a$ the power law is unrenormalized.

If m is not too large (e.g. for large a) we have to take into account the discreteness of m . An instanton solution exists only for $m > \sqrt{E_a/E}$. Since $S_{inst}(z(m), m)$ has always a negative derivative with respect to m at $m \rightarrow \sqrt{E_a/E} + 0$, but for reasonably large values of s the interval of m with negative derivative is much shorter than 1 and hence the optimal hopping length $m_s(E)$ is the smallest integer exceeding $\sqrt{E_a/E}$, which we denote as $[\sqrt{E_a/E}]_{G+}$. To be more realistic we have to take into account the randomness of the impurity distances a_i such that decreasing the field (or the temperature), the current jumps by a factor $\sim e^{-2a_m/\xi_{loc}}$. Clearly, for long wires these jumps will average out.

Finally, we briefly compare the present case of Poissonian strong disorder, $u_{eff}a \gg 1$ with the Gaussian weak disorder, $u_{eff}a \ll 1$ considered in^{10,13,14,18}. In the latter case u and a are sent simultaneously to zero but the quantity $u^2/a \sim \xi_0^{-3} \ll k_F^3$ is assumed to be finite, ξ_0 denotes the bare correlation length. Fluctuations on scales smaller than ξ_0 renormalize $\xi_0 \rightarrow \xi \sim k_F^{-1} (\xi_0 k_F)^{3/(3-2K)}$. At low T the conductivity is of variable range hopping type (8) up to a temperature $T_{loc} = \hbar v/\xi = T_a (u/u_c)^{2/(3-2K)}$ where $u_c \approx k_F (ak_F)^{K-1}$. For higher T there is a direct crossover to $\sigma \sim T^{2-2K(T)}$ where K is now renormalized by disorder fluctuations^{18,19}. This renormalization disappears only at much higher $T_a \sim \hbar v/a$. Both

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weak and strong pinning theories should roughly coincide for $u \rightarrow u_c \approx k_F(ak_F)^{K-1}$ where $T_a \approx T_{1,cr} \approx T_{loc}$ which is indeed the case since $\xi \approx a$. In the strong pinning region ξ continues as $\xi \sim a/s$.

Experimentally, a linear variable range hopping conductivity has been seen in carbon-nanotubes³ and polydiacetylen⁴.

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